ABSTRACT

for the degree Dissertation on the topic "Adequate numerical computation of inverse problem in mathematical geophysics related to ongoing potential fields towards disturbing masses" for the degree Doctor of Philosophy (PhD) in the specialty "8D05401 – Mathematics"
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Relevance of the research topic.

This work is dedicated to solving the inverse problem in geophysics related to the continuation of potential fields towards perturbing masses. This leads to investigating numerical methods for solving the first kind of Fredholm integral equations.

The inverse problem of extending potential fields towards perturbing masses is an important step in interpreting anomalies of valuable minerals. For Kazakhstan, the extraction of valuable minerals and the exploration of new deposits have been relevant tasks for many years. The comprehensive geological exploration of the territory through deep geological and geological-mineralogical mapping is of utmost importance for developing Kazakhstan's geological industry. Additionally, exploring new deposits is a current task instead of depleting old ones.

The depth of geological understanding of deposits serves as the cornerstone for making informed decisions in the effective management of subsoil development. The current state of research in this field reveals a notable fragmentation of information concerning the subsoil, primarily stemming from the discrete nature of its acquisition. Relying on the interpretation of such incomplete data for engineering forecasting can result in strategically flawed and economically inefficient decisions, diminished efficacy in geological exploration, and the occurrence of industrial emergencies.

The uniqueness of this study lies in the development of a software module that operates based on the application of the theory of inverse problems in mathematical geophysics, utilizing geological data from the earth's surface, geophysical measurements, and geochemical analyses as input parameters. This software module, designed for solving the inverse problem of extending potential fields towards disturbing masses, is applied to actual data from a specific mineral deposit.

The purpose of the study is to: formulate and investigate efficient numerical methods for solving the challenge of detecting anomalies in the examination of the spatial distribution of chemical elements in fields.

Research objectives:

- Investigate the formulation of the inverse problem in mathematical geophysics related to ongoing potential fields towards perturbing masses.
- Transform the inverse problem into the solution of a first-kind Fredholm integral equation.

- Explore and compare the effectiveness of approximate methods for solving the first-kind Fredholm integral equation.
- Investigate issues related to the convergence and accuracy of the examined methods.
- Analyze the obtained numerical results and assess the accuracy of the developed numerical methods for solving inverse problems in mathematical geophysics and geochemistry using test examples.
- Attain a numerical solution for a two-dimensional first-kind Fredholm integral equation.
- Develop a software module for a geographic information system based on intelligent anomaly detection methods for hidden deposits, aiming for deep predictive-exploratory modeling of deposits.
- Use the developed module as a suite of applications for digital modeling of a specific deposit to identify concealed objects.

Research methods. In the dissertation, the inverse problem of mathematical geophysics on the continuation of potential fields towards disturbing masses is studied using the apparatus of integral equations and reduced to the first kind of Fredholm integral equation for a large number of different right-hand sides. For an approximate numerical solution of the first kind of Fredholm integral equation, the following methods were used: A.N. Tikhonov's regularization method, a two-stage method based on the theory of conjugate equations, G.N. Pologiy's method, constructive method with "accompaniment", Bubnov-Galerkin method with bases in the form of Legendre wavelets. The method of a priori estimates is used to prove the convergence of projection methods with bases in the form of wavelets on half-intervals.

Scientific novelty. The following new results were obtained in the dissertation:

- Developed an algorithm for the method of conjugate equations for the numerical solution of the task involving the processing of large datasets.
- Formulated an algorithm for implementing the numerical Bubnov-Galerkin method with bases represented by Legendre wavelets.
- Established lemmas regarding the properties of the projection operator in the Galerkin-Bubnov method.
- Proved a lemma for estimating the residual error on semi-intervals.
- Formulated a theorem on the accuracy of the solution in the presence of specified perturbations in the right-hand side and elements of the matrix obtained during the discretization of the first-kind Fredholm integral equation.
- Obtained numerical results for the solution of the first-kind Fredholm integral equation in one-dimensional and two-dimensional cases.
- Developed a mathematical model and numerical methods for solving the problem of anomaly detection in the spatial distribution of chemical elements in deposits.

The main provisions (proven scientific hypotheses and other conclusions that are new knowledge) submitted for defense:

- Developed an algorithm for the method of conjugate equations to numerically address the processing of large datasets.
- Formulated the projection method of Bubnov-Galerkin with bases in the form of Legendre wavelets.
- Derived an a priori error estimate on semi-intervals for the Bubnov-Galerkin method with bases in the form of Legendre wavelets.
- Obtained an estimate of the solution error for the first kind of Fredholm integral equation.
- Attained a numerical solution for a two-dimensional the first kind of Fredholm integral equation.
- Analyzed the obtained numerical results.
- Developed methods for the numerical implementation of inverse problems in mathematical geophysics and geochemistry.
- Developed a software module for a geographic information system based on intelligent anomaly detection methods for hidden deposits, contributing to deep predictive-exploratory modeling of specific deposits.

Theoretical and practical significance of the research. Theoretical research carried out on the topic of the dissertation makes a great contribution to the development of numerical solutions to inverse and ill-posed problems of mathematical geophysics. The practical significance of the work is that the numerical algorithms for solving the first kind of Fredholm integral equation, proposed in this dissertation, are the basis of service software for converting digital data into a geographic information system.

Approbation of work.

The main research results were presented at the following conferences:

- Numerical Functional Analysis - 2021 Conference (Istanbul, Turkey, November 22-24, 2021);
- Modern problems of applied mathematics and information (MPAMIT) – 2021 (Fergana, Uzbekistan November 15–17, 2021);
- VII World Congress of Mathematicians of the Turkic World (TWMS Congress-2023);

Reports were also made at a seminar led by Professor Kanguzhin B.E. at the Department of Mathematics of the Faculty of Mechanics and Mathematics of al-Farabi Kazakh NU and at the seminar of the Department of Mathematics of the International School of Engineering EKTU named after D. Serikbayev.

Publications.

Based on the results of Nurmangalieva's dissertation research, a total of 4 scientific works were published. Including, in publications recommended by Committee of Science of the Ministry of Science and Higher Education of the Republic of Kazakhstan - 1:

1. Temirbekov N.M., Los V.L., Imangaliev E.I., Baygereev D.R., Temirbekova L.N., **Nurmangalieva M.B**. Module of a geographic information system based on numerical modeling of inverse problems of geochemistry using regularizing algorithms // Bulletin of the Abai KazNPU, The series of "physics and mathematical sciences". - Almaty, 2021. - No. 3 (75). - P. 15-28 (on Russian).

in international peer-reviewed journals indexed in the Web of Science database - 3:

1. Temirbekov N., Imangaliyev Y., Baigereyev D., Temirbekova L. **Nurmangaliyeva M.** Numerical simulation of inverse geochemistry problems by regularizing algorithms // Cogent Engineering, Vol.9, Issue 1, 2022, pp.1-21. https://doi.org/10.1080/23311916.2021.2003522
   January - (2022), (WoS Journal Impact Factor JCR(2022) - 1.9, Engineering, Multidisciplinary category quartile - Q2).
2. Temirbekov N., Temirbekova L., **Nurmangaliyeva M.** Numerical solution of the first kind of Fredholm integral equations by projection methods with wavelets as the basis functions // TWMS J. Pure Appl. Math., V.13, N.1, 2022, pp.105-118 http://www.twmsj.az/Files/V.13%20N.1%202022/105-118.pdf
   April - (2022), (WoS Journal Impact Factor JCR (2022)-3.8, category-Mathematics category quartile- Q1).

and 1 article in the collection of materials of international scientific and practical foreign conferences, indexed in the Scopus database:

1. Temirbekov N., Temirbekova L., **Nurmangaliyeva M.**On effective methods of regularization with discretization of integral equations//AIP Conference Proceedings 2781, 020002 (2023) https://doi.org/10.1063/5.0144856
   June - (2023), (Scopus SJR=0,164 (Q4), CiteScore=0.7, Scopus Percentile=15in categoryGeneral Physics and Astronomy).

Structure and scope of work.

The dissertation is written in the form of a manuscript in the Russian language, comprising a title page, table of contents, regulatory references, introduction, four sections, conclusion, and a list of 77 references used. The total volume of the dissertation is 88 pages, including 17 illustrations, 3 tables, and Appendix A.

The introduction outlines the relevance and novelty of the topic, the main goals and objectives of the research proposed for the defense of the position. The introduction provides an overview of publications related to the topic of the dissertation and substantiates the relevance of the research topic.

Section 1 poses the inverse problem of continuation of potential fields based on the concepts of gravity survey and geophysical fields of the Earth. The posed inverse problem is reduced to the first kind of Fredholm integral equation.

Section 2 discusses numerical methods for solving the first kind of Fredholm integral equation: A.N. Tikhonov's regularization method, a two-stage method based on the theory of conjugate equations, G.N. Pologiy's method, constructive method with "accompaniment". A comparative analysis of numerical results was carried out on test problems.

Section 3 describes another numerical method, the Bubnov-Galerkin method with Legendre wavelet bases for solvingthe first kind of Fredholm integral equation. The convergence of the method is proven and a priori estimates are obtained. A numerical experiment was carried out. The use of wavelets to solve The first kind of Fredholm integral equations using the Galerkin method has shown fairly good efficiency. Numerical calculations show that the use of Legendre wavelets as basis functions has a positive effect for the numerical or analytical calculation of integrals in a computational scheme.

Section 4 presents a description of the software module of the geographic information system, which uses intelligent methods to identify hidden deposits and predict their deep location. The functioning of this module is based on the theory of inverse problems of mathematical geophysics, which uses geological data, geophysical measurements and geochemical analyzes as input data. The software module is used to solve the inverse problem of the continuation of potential fields towards the disturbing masses and is applied to real data of a specific mineral deposit.

The main results obtained during the dissertation work are presented,in conclusion.

Appendix A presents the Fortran program code for executing calculations using the Bubnov-Galerkin method with bases represented by Legendre wavelets.